Section 5.4 – Review of Sum and Difference Formulas

$$\sin(\theta+\beta)=\sin\theta\cos\beta+\cos\theta\sin\beta$$

$$\sin(\theta-\beta)=\sin\theta\cos\beta-\cos\theta\sin\beta$$

$$\cos(\theta+\beta)=\cos\theta\cos\beta-\sin\theta\sin\beta$$

$$\cos(\theta-\beta)=\cos\theta\cos\beta+\sin\theta\sin\beta$$

$$\tan(\theta+\beta)=\frac{\tan\theta+\tan\beta}{1-\tan\theta\tan\beta}$$

$$\tan(\theta-\beta)=\frac{\tan\theta-\tan\beta}{1+\tan\theta\tan\beta}$$

Find the EXACT value of the following using sum and difference formulas:

1) $\sin(15^\circ)$

$=\sin(45^\circ-30^\circ)$

$=\sin45^\circ\cos30^\circ-\cos45^\circ\sin30^\circ$

$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$=\frac{\sqrt{6}-\sqrt{2}}{4}$

2) $\tan(75^\circ)$

$=\tan(45^\circ+30^\circ)$

$=\frac{\tan45^\circ+\tan30^\circ}{1-\tan45^\circ\tan30^\circ}=\frac{1+\frac{\sqrt{3}}{3}}{1-1\cdot\frac{\sqrt{3}}{3}}$

$=\frac{3+\sqrt{3}}{3}\cdot\frac{3}{3-\sqrt{3}}=\frac{3+\sqrt{3}}{3-\sqrt{3}}\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)$

$=\frac{9+6\sqrt{3}+3}{9-3}=\frac{12+6\sqrt{3}}{6}=2+\sqrt{3}$

3) $\cos(195^\circ)=-\cos(15^\circ)$

$=-\cos(45^\circ-30^\circ)$

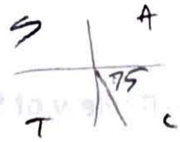
$=-(\cos45^\circ\cos30^\circ+\sin45^\circ\sin30^\circ)$

$=-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$=\frac{-\sqrt{6}-\sqrt{2}}{4}$

4) $\sin\left(\frac{11\pi}{12}\right)=\sin(15^\circ)$

$=\frac{\sqrt{6}-\sqrt{2}}{4}$



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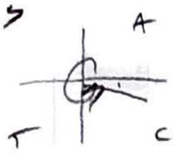
5) $\tan\left(\frac{23\pi}{12}\right)$



$= -\tan(15^\circ)$

$= -\tan(45^\circ - 30^\circ)$

$= -\left(\dfrac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45 \tan 30^\circ}\right)$

$= -\left(\dfrac{1 - \frac{\sqrt{3}}{3}}{1 + 1\cdot\frac{\sqrt{3}}{3}}\right) = \dfrac{\sqrt{3}-3}{3}\cdot\dfrac{3}{\sqrt{3}+3}$

$= \dfrac{\sqrt{3}-3}{\sqrt{3}+3}\left(\dfrac{\sqrt{3}-3}{\sqrt{3}-3}\right) = \dfrac{3-6\sqrt{3}+9}{3-9} = -2+\sqrt{3}$

6) $\cos\left(\frac{19\pi}{12}\right) = \cos 75^\circ = \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

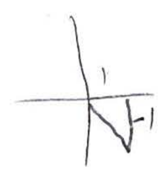
$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Write as sine, cosine, or tangent of an angle. After writing as a single function, find the value: (if possible)

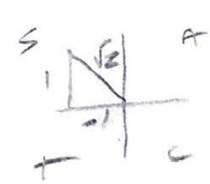
7) $\dfrac{\tan 35^\circ + \tan 10^\circ}{1 - \tan 35^\circ \tan 10^\circ} = \tan 45^\circ = 1$

☆ 8) $\dfrac{\tan 68^\circ - \tan 115^\circ}{1 + \tan 68^\circ \tan 115^\circ} = \tan(68 - 115) = \tan(-47^\circ)$



9) $\sin(72)\cos(12) - \cos(72)\sin(12) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

10) $\cos 15^\circ \cos 120^\circ - \sin 15^\circ \sin 120^\circ = \cos(135^\circ) = -\frac{\sqrt{2}}{2}$



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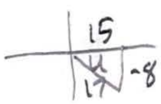
11) $\dfrac{\tan(100)+\tan(20)}{1-\tan(100)\tan(20)}$ $= \tan(100+20) = \tan 120 = -\tan 60 = -\sqrt{3}$

12) $\sin 90°\cos 45° - \cos 90°\sin 45°$ $= \sin(90-45) = \sin 45° = \frac{\sqrt{2}}{2}$

13) $\cos(94°)\cos(18°)+\sin(94°)\sin(18°)$ $= \cos(94-18) = \cos 76°$

14) Find the EXACT value of the trig function given that

$$\sin u = \frac{-8}{17} \quad \cos v = \frac{4}{5} \text{ and both angles are in Quadrant IV}$$



a) $\tan(u-v)$

$= \dfrac{\tan u - \tan v}{1+\tan u \tan v}$

$= \dfrac{\frac{-8}{15} - \frac{-3}{4}}{1+\frac{-8}{15}\cdot\frac{-3}{4}}$

$= \dfrac{\frac{-32+45}{60}}{\frac{60+24}{60}} = \frac{13}{84}$

b) $\cos(u+v)$

$= \cos u \cos v - \sin u \sin v$

$= \frac{15}{17}\cdot\frac{4}{5} - \frac{-8}{17}\cdot\frac{-3}{5}$

$= \frac{60}{85} - \frac{24}{85} = \frac{36}{85}$

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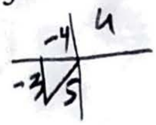
~~13~~) Find the **exact value** of the trig function given that

$$\sin u = \frac{-3}{5} \text{ and } \cos v = \frac{-5}{13} \text{ where both u and v are in Quadrant III.}$$



a) $\sin(u-v)$

$= \sin u \cos v - \cos u \sin v$

$= \frac{-3}{5} \cdot \frac{-5}{13} - \frac{-4}{5} \cdot \frac{-12}{13}$

$= \frac{15}{65} - \frac{48}{65} = -\frac{33}{65}$

b) $\cos(u-v)$

$= \cos u \cos v + \sin u \sin v$

$= -\frac{4}{5} \cdot \frac{-5}{13} + \frac{-3}{5} \cdot \frac{-12}{13}$

$= \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$

c) $\tan(u+v)$

$$= \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4}\left(\frac{12}{5}\right)} = \frac{\frac{15+48}{20}}{\frac{20-36}{20}} = -\frac{63}{16}$$

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16) Solve the following equations in the domain $[0, 2\pi)$

a) $2(\sin 2x \cos x + \cos 2x \sin x) = \sqrt{2}$

$$\sin(2x + x) = \frac{\sqrt{2}}{2}$$

$$\sin 3x = \frac{\sqrt{2}}{2}$$

$$u = 3x$$

$$\sin u = \frac{\sqrt{2}}{2}$$

$$u = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$3x = \frac{\pi}{4} \qquad 3x = \frac{3\pi}{4}$$

$$x = \frac{\pi}{12}, \frac{\pi}{4}$$

b) $2\cos 2x \cos 3x - 2\sin 2x \sin 3x = 1$

$$2(\cos(2x + 3x)) = 1$$

$$\cos 5x = \frac{1}{2}$$

$$5x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{\pi}{15}, \frac{\pi}{3}$$